MATHEMATICS SPECIALIST 3CD

Section One:
Calculator-free

Please place your student identification label in this box

SOLUTIONS

Time allowed for this section

Reading time before commencing work: five minutes
Working time for this section: forty minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available
Section One: Calculator-free	6	6	40	35
Section Two: Calculator-assumed	11	11	80	65
			Total	100

Instructions to candidates

1. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

2. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

3. It is recommended that you **do not use pencil**, except in diagrams.

Section One: Calculator-free **(35 Marks)**

This section has **six (6)** questions. Answer **all** questions. Write your answers in the space provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

Working time: 40 minutes.

Question 1 **(4 marks)**

The graphs of r = 5, $\theta = \dfrac{\pi}{3}$ and $r = \dfrac{5}{\pi}\theta$ for $0 \le \theta \le 2\pi$ are drawn below
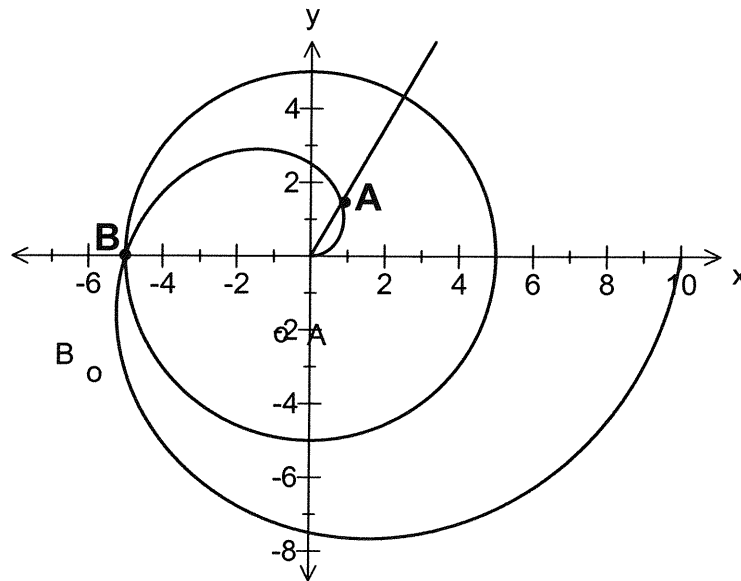
(a) Write down the polar coordinates of

   (i) Point A $\left(\dfrac{5}{3}, \dfrac{\pi}{3}\right)$ ✓ (1 mark)

   (ii) Point B $\left(5, \pi\right)$ ✓ (1 mark)

(b) Find the EXACT distance between A and B. (2 marks)

$$AB^2 = \left(\frac{5}{3}\right)^2 + 5^2 - 2 \cdot \frac{5}{3} \cdot 5 \cdot \cos\frac{2\pi}{3} \checkmark$$

$$= \frac{250}{9} + \frac{25}{3}$$

$$\therefore AB = \sqrt{\frac{325}{9}} \checkmark = \frac{5\sqrt{13}}{3}$$

See next page

Question 2   **(7 marks)**

Determine $\dfrac{dy}{dx}$ for each of the following. Do not simplify your answers

(a) $y = \ln\left(\dfrac{1}{1-x^2}\right)$ **(2 marks)**

$$= -\ln(1-x^2) \quad \checkmark$$

$$\frac{dy}{dx} = \frac{2x}{1-x^2} \quad \checkmark$$

(b) $y = x^3 \sin^2(5x)$ **(2 marks)**

$$\frac{dy}{dx} = x^3 \cdot 2\sin 5x \cos 5x \cdot 5 + 3x^2 \sin^2 5x \quad \checkmark \quad \checkmark$$

(c) $\log x^y = 3x$ **(3 marks)**

$$y \log x = 3x$$

$$y \frac{\ln x}{\ln 10} = 3x \quad \checkmark$$

$$y \ln x = 3 \ln 10 \, x$$

$$\frac{dy}{dx} \cdot \ln x + y \cdot \frac{1}{x} = 3 \ln 10 \quad \checkmark$$

$$\frac{dy}{dx} = \frac{3 \ln 10 - \frac{y}{x}}{\ln x} \quad \checkmark$$

Question 3   **(7 marks)**

Find the following indefinite integrals.

(a) $\int 12e^{2x}(1-e^{2x})\,dx$ (2 marks)

$$= \frac{-6\left(1-e^{2x}\right)^2}{2} + c$$

$$= -3\left(1-e^{2x}\right)^2 + c$$

(b) $\int sinx\,(1-sinx)\,dx$ (3 marks)

$$= \int sinx - sin^2x\,dx$$

$$= -cosx - \int \frac{1-cosx}{2}\,dx$$

$$= -cosx - \frac{1}{2}x + \frac{sin2x}{4} + c$$

(c) $\int \frac{6x^2-12}{x^3-6x+3}\,dx$ (2 marks)

$$= 2\ln\left|x^3-6x+3\right| + c$$

Question 4   **(7 marks)**

(a)   Find the exact value of the definite integral below:

$$\int_0^3 \frac{x}{\sqrt{16-x^2}}\, dx \qquad \text{Use the substitution} \quad u = 16 - x^2 \qquad \text{(4 marks)}$$

$$= \int_{16}^7 \frac{x}{\sqrt{u}} \cdot \frac{1}{-2x}\, du$$

$$\frac{du}{dx} = -2x$$

$$\therefore dx = \frac{1}{-2x}\, du$$

$$x = 3 \quad u = 7$$
$$x = 0 \quad u = 16$$

$$= -\frac{1}{2}\left[2u^{\frac{1}{2}}\right]_{16}^7$$

$$= \left[-\sqrt{u}\right]_{16}^7$$

$$= -\sqrt{7} - (-\sqrt{16})$$

$$= 4 - \sqrt{7}$$

(b)   Show that $5^{(2 + \log_5 3)} = 75$ **(3 marks)**

$$\text{let} \quad y = 5^{2 + \log_5 3}$$

$$\therefore \ln y = \ln 5^{2 + \log_5 3} \qquad \checkmark$$

$$= (2 + \log_5 3)\ln 5$$

$$= 2\ln 5 + \frac{\ln 3}{\ln 5}\cdot \ln 5$$

$$= \ln 25 + \ln 3 \qquad \checkmark$$
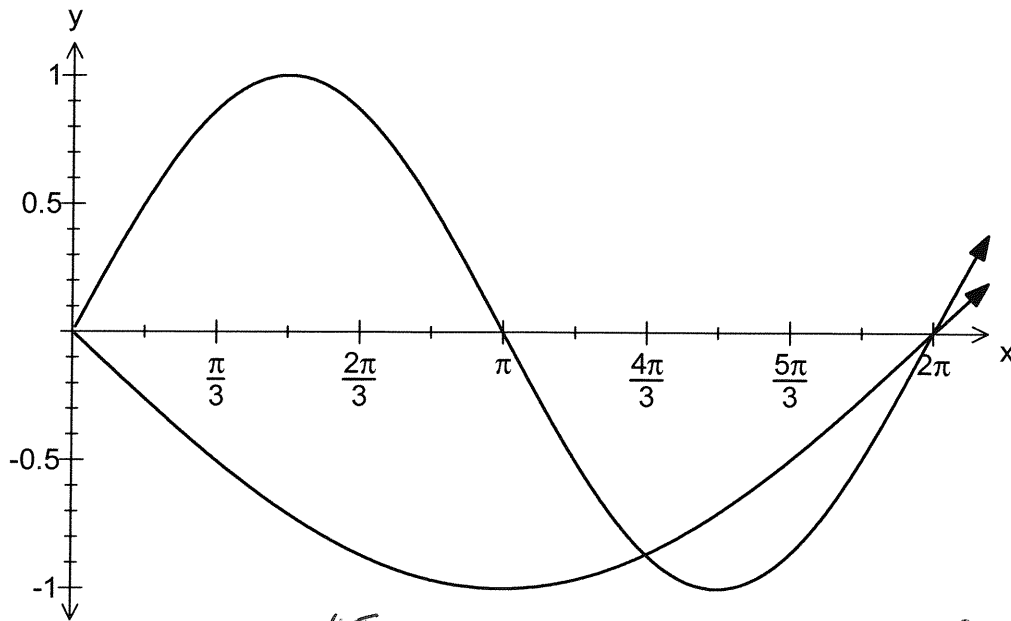
$$\therefore \ln y = \ln 75$$

$$\therefore y = 75 \qquad \checkmark$$

$$\therefore 5^{2 + \log_5 3} = 75$$

Question 5 (4 marks)

Determine the area bounded by the curves $f(x) = \sin x$ and $g(x) = -\sin\left(\dfrac{x}{2}\right)$ for

$0 \le x \le 2\pi$.

$$\text{Area} = \int_{0}^{\frac{4\pi}{3}} \sin x - \left(-\sin \frac{x}{2}\right) dx + \int_{\frac{4\pi}{3}}^{2\pi} -\sin \frac{x}{2} - \sin x \, dx$$

$$= \int_{0}^{\frac{4\pi}{3}} \sin x + \sin \frac{x}{2} \, dx - \int_{\frac{4\pi}{3}}^{2\pi} \sin x + \sin \frac{x}{2} \, dx$$

$$= \left[-\cos x - 2\cos \frac{x}{2}\right]_{0}^{\frac{4\pi}{3}} - \left[-\cos x - 2\cos \frac{x}{2}\right]_{\frac{4\pi}{3}}^{2\pi}$$

$$= \left(-\cos \frac{4\pi}{3} - 2\cos \frac{4\pi}{6}\right) - \left(-\cos 0 - 2\cos 0\right)$$

$$- \left(-\cos 2\pi - 2\cos \pi\right) + \left(-\cos \frac{4\pi}{3} - 2\cos \frac{4\pi}{6}\right)$$

$$= \left(\tfrac{1}{2} + 1\right) - \left(-1 - 2\right) - \left(-1 + 2\right) + \left(\tfrac{1}{2} + 1\right)$$

$$= 5$$

Question 6   **(6 marks)**

(a) Find $\dfrac{d}{dx}(x\cos x)$, showing full working.   (2 marks)

$$\frac{d}{dx}\left(x\cos x\right) = x(-\sin x) + \cos x$$

$$= \cos x - x\sin x$$

(b) **Hence**, evaluate the following definite integrals, giving exact values.

(i) $\displaystyle\int_0^\pi \cos x - x\sin x \; dx$ (2 marks)

$$= \left[x\cos x\right]_0^\pi$$

$$= \pi\cos\pi - 0\cdot\cos 0$$

$$= -\pi$$

(ii) $\displaystyle\int_0^\pi x\sin x \; dx$ (2 marks)

$$\int_0^\pi \cos x - x\sin x \; dx = \left[x\cos x\right]_0^\pi$$

$$\therefore \int_0^\pi x\sin x \; dx = \int_0^\pi \cos x \; dx - \left[x\cos x\right]_0^\pi$$

$$= \left[\sin x\right]_0^\pi - (-\pi)$$

$$= \sin\pi - \sin 0 + \pi$$

$$= \pi$$

End of questions